

# Accurate Nucleon-Nucleon Potential Based upon Chiral Perturbation Theory

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We present an accurate nucleon-nucleon ( $NN$ ) potential based upon chiral effective Lagrangians. The model includes one- and two-pion exchange contributions up to chiral order three. We show that a quantitative fit of the  $NN$   $D$ -wave phase shifts requires contact terms (which represent the short range force) of order four. Within this framework, the  $NN$  phase shifts below 300 MeV lab. energy and the properties of the deuteron are reproduced with high-precision. This chiral  $NN$  potential represents a reliable starting point for testing the chiral effective field theory approach in exact few-nucleon and microscopic nuclear many-body calculations. An important implication of the present work is that the chiral  $2\pi$  exchange at order four is of crucial interest for future chiral  $NN$  potential development.

One of the most fundamental problems of nuclear physics is to derive the force between two nucleons from first principles. A great obstacle for the solution of this problem has been the fact that the fundamental theory of strong interaction, QCD, is nonperturbative in the low-energy regime characteristic for nuclear physics. The way out of this dilemma is the effective field theory concept which recognizes different energy scales in nature. Below the chiral symmetry breaking scale,  $\Lambda_\chi \approx 1$  GeV, the appropriate degrees of freedom are pions and nucleons interacting via a force that is governed by the symmetries of QCD, particularly, (broken) chiral symmetry.

The derivation of the nuclear force from chiral effective field theory was initiated by Weinberg [1] and pioneered by Ordóñez [2] and van Kolck [3,4]. Subsequently, many groups got involved in the subject [5–10]. As a result, efficient methods for deriving the nuclear force from chiral Lagrangians have emerged. Also, the quantitative nature of the chiral  $NN$  potential has improved [10]. Nevertheless, even the currently ‘best’ chiral  $NN$  potentials are too inaccurate to serve as a reliable input for exact few-nucleon calculations or microscopic nuclear many-body theory.

The time has come to put the chiral approach to a real test in microscopic nuclear structure physics. Conclusive results can, however, be produced only with a 100% quantitative  $NN$  potential based upon chiral Lagrangians. For this reason, we have embarked on a program to develop a  $NN$  potential that is based upon chiral effective field theory and reproduces the  $NN$  data with about that same quality as the high-precision  $NN$  potentials constructed in the 1990’s [11–14].

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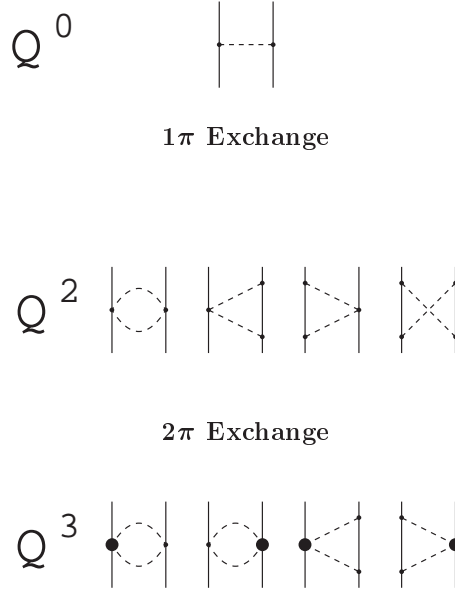


FIG. 1. The most important irreducible one- and two-pion exchange contributions to the  $NN$  interaction up to order  $Q^3$ . Vertices denoted by small dots are from  $\hat{\mathcal{L}}_{\pi N}^{(1)}$ , while large dots refer to  $\hat{\mathcal{L}}_{\pi N, \text{ct}}^{(2)}$ .

Starting point for the derivation of the  $NN$  interaction is an effective chiral  $\pi N$  Lagrangian which is given by a series of terms of increasing chiral dimension [15],

$$\mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \dots, \quad (1)$$

where the superscript refers to the number of derivatives or pion mass insertions (chiral dimension) and the ellipsis denotes terms of chiral order four or higher.

We will apply the heavy baryon (HB) formulation of chiral perturbation theory [16] in which the relativistic Lagrangian is subjected to an expansion in terms of powers of  $1/M_N$  (kind of a nonrelativistic expansion), the lowest order of which is

$$\hat{\mathcal{L}}_{\pi N}^{(1)} = \bar{N} \left( iD_0 - \frac{g_A}{2} \vec{\sigma} \cdot \vec{u} \right) N \quad (2)$$

$$\approx \bar{N} \left[ i\partial_0 - \frac{1}{4f_\pi^2} \boldsymbol{\tau} \cdot (\boldsymbol{\pi} \times \partial_0 \boldsymbol{\pi}) - \frac{g_A}{2f_\pi} \boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \boldsymbol{\pi} \right] N + \dots, \quad (3)$$

where we use the notation of Ref. [16]. For the parameters that occur in the leading order Lagrangian, we apply  $M_N = 938.919$  MeV,  $m_\pi = 138.04$  MeV,  $f_\pi = 92.4$  MeV, and  $g_A = g_{\pi NN} f_\pi / M_N = 1.29$ , which is equivalent to  $g_{\pi NN}^2 / 4\pi = 13.67$ .

The HB projected Lagrangian at order two is most conveniently broken up into two pieces,

$$\hat{\mathcal{L}}_{\pi N}^{(2)} = \hat{\mathcal{L}}_{\pi N, \text{fix}}^{(2)} + \hat{\mathcal{L}}_{\pi N, \text{ct}}^{(2)}, \quad (4)$$

with

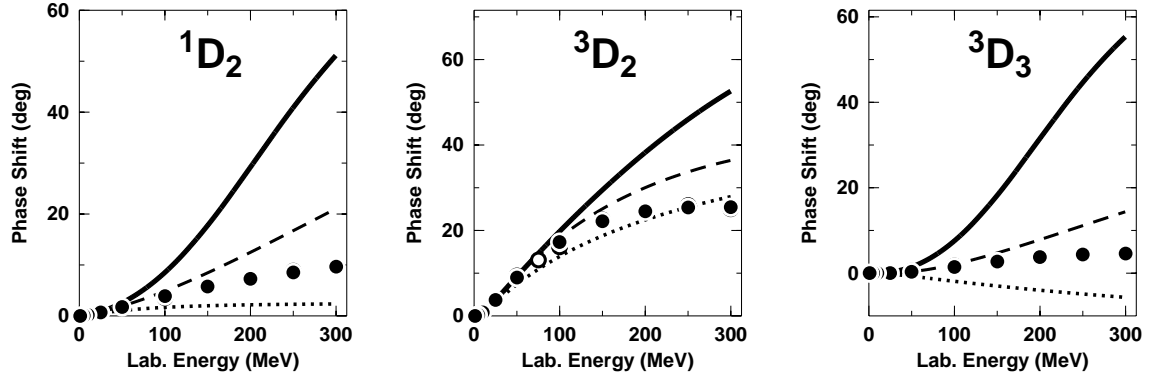


FIG. 2.  $D$ -wave phase shifts of  $NN$  scattering. The predictions by the chiral model displayed in Fig. 1 are shown by the solid line and the ones by the Bonn  $\pi + 2\pi$  model [18] by the dashed curve. The dotted line is OPE. Solid dots represent the Nijmegen multi-energy  $np$  analysis [19] and open circles the VPI/GWU analysis [20].

$$\hat{\mathcal{L}}_{\pi N, \text{fix}}^{(2)} = \bar{N} \left[ \frac{1}{2M_N} \vec{D} \cdot \vec{D} + i \frac{g_A}{4M_N} \{ \vec{\sigma} \cdot \vec{D}, u_0 \} \right] N \quad (5)$$

and

$$\hat{\mathcal{L}}_{\pi N, \text{ct}}^{(2)} = \bar{N} \left[ 2c_1 m_\pi^2 (U + U^\dagger) + \left( c_2 - \frac{g_A^2}{8M_N} \right) u_0^2 + c_3 u_\mu u^\mu + \frac{i}{2} \left( c_4 + \frac{1}{4M_N} \right) \vec{\sigma} \cdot (\vec{u} \times \vec{u}) \right] N. \quad (6)$$

Note that  $\hat{\mathcal{L}}_{\pi N, \text{fix}}^{(2)}$  is created entirely from the HB expansion of the relativistic  $\mathcal{L}_{\pi N}^{(1)}$  and thus has no free parameters (“fixed”), while  $\hat{\mathcal{L}}_{\pi N, \text{ct}}^{(2)}$  is dominated by  $\pi N$  contact terms proportional to the  $c_i$  parameters, besides some small  $1/M_N$  corrections. The parameters  $c_i$  are known as low-energy constants (LECs) and must be determined empirically from fits to  $\pi N$  data. We use the values determined by Büttiker and Meißner [17] which are (in units of  $\text{GeV}^{-1}$ )  $c_1 = -0.81$ ,  $c_3 = -4.70$ , and  $c_4 = 3.40$  ( $c_2$  will not be needed).

The  $\pi N$  Lagrangian is the crucial ingredient for the evaluation of the pion-exchange contributions to the  $NN$  interaction. Since we are dealing here with a low-energy effective theory, it is appropriate to analyze the contributions in terms of powers of small momenta:  $(Q/\Lambda_\chi)^\nu$ , where  $Q$  is a generic momentum or a pion mass and  $\Lambda_\chi \approx 1 \text{ GeV}$  is the chiral symmetry breaking scale. This procedure has become known as power counting. For the pion-exchange diagrams relevant to our problem, the power  $\nu$  of a diagram is determined by the simple formula

$$\nu = 2l + \sum_j (d_j - 1), \quad (7)$$

where  $l$  denotes the number of loops in the diagram,  $d_j$  the number of derivatives involved in vertex  $j$ , and the sum runs over all the vertices of the diagram.

The most important irreducible one-pion exchange (OPE) and two-pion exchange (TPE) contributions to the  $NN$  interaction up to order  $Q^3$  are shown in Fig. 1; they have been evaluated by Kaiser *et al.* [6] using covariant perturbation theory and dimensional regularization. One- and two-pion exchanges are known to describe  $NN$  scattering in peripheral

partial waves. In  $G$  and higher partial waves (orbital angular momentum  $L \geq 4$ ), there is good agreement between the chiral and conventional [18]  $2\pi$  model as well as the empirical phase shifts [19,20]. The agreement deteriorates when proceeding to lower  $L$ . While in  $F$  waves the agreement between the chiral model and the empirical phase shifts is still fair, substantial discrepancies emerge in  $D$  waves, Fig. 2, where the chiral  $2\pi$  exchange is far too attractive—a fact that has been noticed before [6,7].

To control the  $D$  (and lower) partial waves, we need (repulsive) short-range contributions. In the conventional meson model [18], these are created by the exchange of heavy mesons (notably, the  $\omega$  meson). In chiral perturbation theory ( $\chi$ PT), heavy mesons have no place and the short-range force is parametrized in terms of contact potentials, which are organized by powers of  $Q$ . If  $Q$  is, e. g., a momentum transfer, i. e.,  $\vec{Q} = \vec{p}' - \vec{p}$ , where  $\vec{p}$  and  $\vec{p}'$  are the CM nucleon momenta before and after scattering, respectively, and  $\theta$  is the scattering angle, then, for even  $\nu$ ,

$$\vec{Q}^\nu \sim (\cos \theta)^m \quad \text{with} \quad m \leq \frac{\nu}{2}. \quad (8)$$

Partial-wave decomposition for orbital-angular momentum  $L$  yields,

$$\int_{-1}^{+1} \vec{Q}^\nu P_L(\cos \theta) d\cos \theta \neq 0 \quad \text{for} \quad L \leq \frac{\nu}{2}, \quad (9)$$

where  $P_L$  is a Legendre polynomial. The conclusion is that for non-vanishing contributions in  $D$  waves ( $L = 2$ ),  $\nu = 4$  is required. *This one important message that we like to convey in this letter.* Based upon invariance considerations, there are a total of 24 contact terms up to order  $Q^4$ , which we all include in our model. The parameters of these terms have to be natural, but are otherwise unconstrained and, thus, represent essentially free parameters.

To describe  $NN$  scattering, we start from the Bethe-Salpeter (BS) equation [21] which reads in operator notation

$$\mathcal{T} = \mathcal{V} + \mathcal{V} \mathcal{G} \mathcal{T} \quad (10)$$

with  $\mathcal{T}$  the invariant amplitude for the two-nucleon scattering process,  $\mathcal{V}$  the sum of all connected two-particle irreducible diagrams, and  $\mathcal{G}$  the relativistic two-nucleon propagator. The BS equation is equivalent to a set of two equations:

$$\mathcal{T} = \bar{V} + \bar{V} g \mathcal{T} \quad (11)$$

$$\bar{V} = \mathcal{V} + \mathcal{V} (\mathcal{G} - g) \bar{V} \quad (12)$$

$$\approx \mathcal{V} + \mathcal{V}_{\text{OPE}} (\mathcal{G} - g) \mathcal{V}_{\text{OPE}} \quad (13)$$

where the last line states the approximation we are using, exhibiting the way we treat the  $2\pi$  box diagram. This treatment avoids double counting when  $\bar{V}$  is iterated in the scattering equation and is also consistent with the calculations of Ref. [6]. For the relativistic three-dimensional propagator  $g$ , we choose the one proposed by Blankenbecler and Sugar [22] (BbS) which has the great practical advantage that the OPE (and the entire potential) becomes energy-independent. Thus, we do not need the rather elaborate formalism of unitary transformations [10] to generate energy-independence of the potential.

Our full chiral  $NN$  potential  $\bar{V}$  is defined by

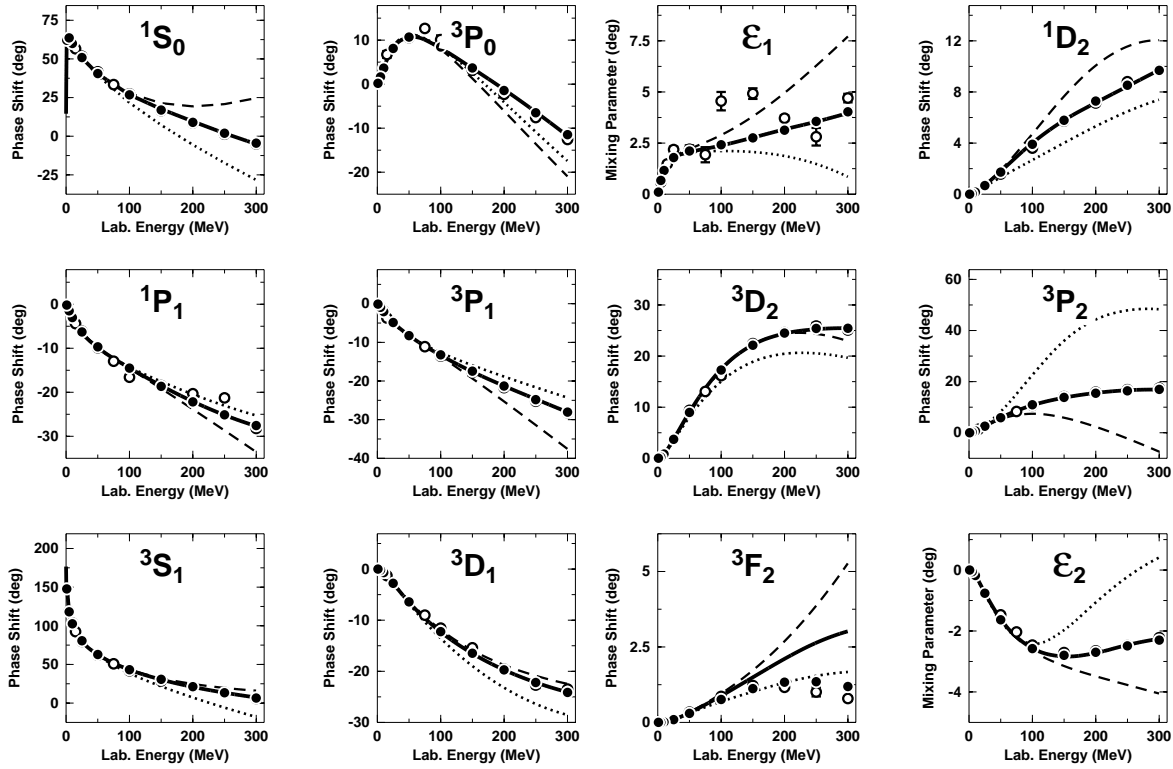


FIG. 3. Phase shifts for  $J \leq 2$ . The solid line is the result from our chiral  $NN$  potential, while the dotted and dashed lines are the predictions by two chiral models developed by Epelbaum *et al.* [10] (NLO and NNLO, respectively). Solid dots and open circles represent phase shift analyses explained in the caption of Fig. 2.

$$\bar{V}(\vec{p}', \vec{p}) \equiv \left\{ \begin{array}{c} \text{sum of irreducible} \\ \pi + 2\pi \text{ contributions} \end{array} \right\} + \text{contacts}, \quad (14)$$

where the first term on the r.h.s. is given by Eq. (13) with  $\mathcal{V}$  containing essentially the diagrams of Fig. 1. This potential satisfies the relativistic BbS equation, Eq. (11). If we define now,

$$V(\vec{p}', \vec{p}) \equiv \sqrt{\frac{M_N}{E_{p'}}} \bar{V}(\vec{p}', \vec{p}) \sqrt{\frac{M_N}{E_p}} \approx \left( 1 - \frac{p'^2 + p^2}{4M_N^2} \right) \bar{V}(\vec{p}', \vec{p}) \quad (15)$$

with  $E_p \equiv \sqrt{M_N^2 + p^2}$ , then  $V$  satisfies the usual, nonrelativistic Lippmann-Schwinger (LS) equation.

Iteration of  $V$  in the LS equation requires cutting  $V$  off for high momenta to avoid infinities. Therefore, we regularize  $V$  in the following way:

$$V(\vec{p}', \vec{p}) \longmapsto V(\vec{p}', \vec{p}) e^{-(p'/\Lambda)^{2n}} e^{-(p/\Lambda)^{2n}} \quad (16)$$

$$\approx V(\vec{p}', \vec{p}) \left\{ 1 - \left[ \left( \frac{p'}{\Lambda} \right)^{2n} + \left( \frac{p}{\Lambda} \right)^{2n} \right] + \dots \right\}, \quad (17)$$

where the last equation is to indicate that the exponential cutoff does not affect the order to which we are calculating, but introduces contributions beyond that order. For the contact

TABLE I. Two- and three-nucleon low-energy data.

	Idaho-A <sup>a</sup>	Idaho-B <sup>a</sup>	CD-Bonn [14]	AV18 [12]	Empirical <sup>b</sup>
<b>Low-energy <math>np</math> scattering</b>					
$^1S_0$ scattering length (fm)	-23.75	-23.75	-23.74	-23.73	-23.74(2)
$^1S_0$ effective range (fm)	2.70	2.70	2.67	2.70	2.77(5)
$^3S_1$ scattering length (fm)	5.417	5.417	5.420	5.419	5.419(7)
$^3S_1$ effective range (fm)	1.750	1.750	1.751	1.753	1.753(8)
<b>Deuteron properties</b>					
Binding energy (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575(9)
Asympt. $S$ state (fm <sup>-1/2</sup> )	0.8846	0.8846	0.8846	0.8850	0.8846(9)
Asympt. $D/S$ state	0.0256	0.0255	0.0256	0.0250	0.0256(4)
Deuteron radius (fm)	1.9756 <sup>c</sup>	1.9758 <sup>c</sup>	1.970 <sup>c</sup>	1.971 <sup>c</sup>	1.9754(9) <sup>d</sup>
Quadrupole moment (fm <sup>2</sup> )	0.281 <sup>e</sup>	0.284 <sup>e</sup>	0.280 <sup>e</sup>	0.280 <sup>e</sup>	0.2859(3)
$D$ -state probability (%)	4.17	4.94	4.85	5.76	
<b>Triton binding (MeV)</b>	8.14	8.02	8.00	7.62	8.48

<sup>a</sup>Chiral NN potential of the present work.

<sup>b</sup>For references concerning the empirical data, see Tables XIV and XVIII of Ref. [14].

<sup>c</sup>With meson-exchange current (MEC) and relativistic corrections [23].

<sup>d</sup>Reference [24].

<sup>e</sup>Including MEC and relativistic corrections in the amount of 0.010 fm<sup>2</sup> [25].

terms, we use partial wave dependent cutoff parameters  $\Lambda \approx 0.4 - 0.5$  GeV which brings the total number of parameters in our chiral  $NN$  model up to 46. At first glance, this may appear to be a large number. Note, however, that the Nijmegen phase shift analysis [19] and the high-precision potentials [11–14] developed in the 1990’s carry between 40 and 50 parameters. Thus, the number of parameters needed for a quantitative chiral  $NN$  model is just about the same as for meson models. Since the chiral model has less predictive power than the meson model this should not be unexpected.

In Fig. 3, we show the phase shifts of neutron-proton ( $np$ ) scattering for lab. energies below 300 MeV and partial waves with  $J \leq 2$ . The solid line represents the result from the chiral  $NN$  potential developed in the present work. *The reproduction of the empirical phase shifts by our model is excellent.* For comparison, we also show the phase shift predictions by two chiral models recently developed by Epelbaum *et al.* [10] (dotted and dashed curves in Fig. 3). Also the effective range parameters in  $S$  waves agree accurately with the empirical values, as well as the deuteron parameters (see Table I). We note that our present chiral potential is charge-independent and adjusted to the  $np$  data.

Due to the very quantitative nature of this new chiral  $NN$  potential, it represents a reliable and promising starting point for exact few-body calculations and microscopic nuclear many-body theory.

A crucial finding of our investigation is that contact terms of order four are required for a quantitative  $NN$  model. The basic ideas of  $\chi$ PT may then suggest that—for reasons of consistency—the chiral  $2\pi$  exchange contribution should also be included up to order four. Therefore, an implication of the present work is that the chiral  $2\pi$  exchange at order four [26] will be important for further chiral  $NN$  potential development.

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